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LAMINAR FULLY DEVELOPED FLOW IN PERIODICALLY CONVERGING-DIVERGING MICROTUBES

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ABSTRACT

Laminar fully developed flow and pressure drop in linearly varying cross -section converging-diverging microtubes have been is investigated in this work. These microtubes are formed from a series of converging-diverging modules. An analytical model is developed for frictional flow resistance assuming parabolic axial velocity profile in diverging and converging sections. Frictional flow resistance is found to be only a function of the geometrical parameters. To validate the model, a numerical study is carried out for the Reynolds number ranging from 0.01 to 100, for various taper angles and maximum-minimum radius ratios ranging from 0.5 to 1. Comparisons between the model and numerical results show that the proposed model predicts the axial velocity and the flow resistance accurately. As expected, the flow resistance is found to be effectively independent of the Reynolds number from numerical results. Parametric study shows that the effect of radius ratio is more significant than the taper angle. It is also observed that for small taper angles, flow resistance can be determined accurately by applying the local Poiseuille flow approximation.

Nomenclature

$a(z)$ = radius of tube, m
a_0 = maximum radius of tube, m
a_1 = minimum radius of tube, m
A_c = cross-sectional area, m^2
L = half of module length, m
m = slope of tube wall, $[-]$
Q = volumetric flow rate, m^3/s
r, z = cylindrical coordinate, m
Re = Reynolds number, $2\rho u_{m,0} a_0/\mu$
R_f = frictional resistance, $m^{-1}s^{-1}$
R_f^* = normalized flow resistance, $R_f/R_{f,0}$
u_m = mean fluid axial velocity, m/s
u, v = velocity in z and r directions, m/s

Greek

β = $a_0/a(z)$
η = $r/a(z)$
ε = a_1/a_0
ρ = fluid density, kg/m^3
μ = fluid viscosity, $kg/m.s$
ϕ = angle of tube wall, $[-]$
ΔP = pressure gradient, Pa

1 INTRODUCTION

Advances in microfabrication make it possible to build microchannels with small characteristic length, in order of micrometers. Micro- and minichannels show promising potential for being incorporated in a wide variety of unique, compact, and efficient cooling applications such as in microelectronic devices. These micro heat exchangers or heat sinks feature extremely high heat transfer surface area per unit volume ratios, high heat transfer coefficients, and low thermal resistances. In biological and life sciences, microchannels are used widely for analyzing biological materials such as proteins, DNA, cells, embryos and chemical reagents. Various microsystems such as micro-heat

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sinks, micro-biochips, micro-reactors and micro-nozzles have been developed in recent years. Since microchannels are usually integrated in these microsystems, it is important to know the characteristics of the fluid flow in these microchannels for better design of various micro-flow devices.

Microchannels are normally produced by either precision machining or chemical etching or recently soft lithography in PDMS. Together with new methods of fabrication, it is possible to expliot certain fundamental differences between the physical properties of fluids moving in large channels and those travelling through micrometre-scale channels [Whiteside]. Using these methods, the channel cross-section would be normally rectangular. Aslo since microchannels length is normally long, the main part of the flow is fully developed.

In recent years, a large number of papers have reported pressure drop data for laminar fully developed flow of liquids in microchannels with various cross-sections. However, published results are often inconsistent. Peng and Peterson et al. [Xu references] experimentally studied friction flow and heat transfer characteristics of water flow through microchannels with hydraulic diameters ranging from 130 μm to 340 μm. They used precision machining on stainless steel substrate method. Their experimental results for flow friction showed that significant deviation from the characteristics of the macro-size channel flow were observed. Their results indicated that unstabilities in the flow started at the Reynolds number of 200 to 700. Mala and Li [9 of Xu] experimentally investigated the water flow in circular cross-section microchannels with diameters ranging from 50 to 254 μm. Their experimental results were presented at the Reynolds number ranging from 100 to 2000. They showed that their results were significantly deviated from the predictions of the conventional theory for microtubes with diameter smaller than 150 μm. their experimental results were larger than those predicted by the conventional theory. They interpreted their anomalous phenomena as either an early transition from laminar flow to turbulent flow or due to surface roughness effects. Pfahler et al. [2, 3 Xu] presented experimental investigations of the fluid flow in microchannels. In their study, nitrogen and helium gases, isopropyl liquid and silicone oil, were used. The microchannels were fabricated on silicon wafer by an etching method. Different phenomena for different fluids were observed. They found that in channels with relatively large cross-sections, the fluid roughly behaved in accordance with the Navier-Stokes predictions, and as the channel depth decreased, there appeared to be a critical channel size where the general behavior of the experimental observations deviates from the predictions.

- Literature review (Experimental)

- Pressure measrument issues: like size, inlet and outlet effects, uncertainty.

2 THEORETICAL MODELING

- Literature review

-Bahrami's Model: Assumptions, saint veron concept, we are going to compar our results with that.

Table 1. INPUT PARAMETERS FOR TWO TYPICAL MICROTUBES.

Parameter	Value
a_0	500 μm
Re	10
$Case1$	
$\phi = 7^\circ$	
$0.5 < \varepsilon < 1$	
$Case2$	
$2^\circ < \phi < 15^\circ$	
$\varepsilon = 0.8$	

3 EXPERIMENTAL

- Microchannel fabrication: Table of sizes

- Experimental set up: figure

- Microchannels cross section picture (Kandlikar can help)

- Experimental data analysis (relationsships for f and ...)

-uncertainty (kandlikar)

4 RESULTS AND DISCUSSION

5 SUMMARY AND CONCLUSIONS

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